THE FOUCAULT PENDULUM IN THE UNITED NATIONS BUILDING IN NEW YORK

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Among the gifts presented by member states for embellishing the United Nations building in New York is a large Foucault pendulum from the Netherlands. The pendulum, which is suspended in the entrance hall leading to the main Assembly Hall, continuously demonstrates by its movement that the earth is in rotation.

The article below briefly describes the suspension and driving systems of this pendulum. The design was based on the requirement that the pendulum should function well for a very long time without supervision and maintenance.

On 7 December 1955, Mr. H. Luns, Dutch Foreign Minister, presented to the Chairman of the United Nations General Assembly a Foucault pendulum on behalf of the people of the Netherlands. The pendulum is suspended above the central column of the staircase in the entrance hall of the United Nations building in New York (see *fig. 1*). The pendulum was specially designed in the Research Laboratory of N.V. Philips' Gloeilampenfabrieken, the guiding consideration being that the pendulum should function uninterruptedly for many years without requiring supervision or maintenance.

A Foucault pendulum is in principle merely a weight (which we shall henceforth refer to as the "bob") suspended by a long wire, swinging in a vertical plane. Owing to the manner of suspension the plane of swing is not, however, fixed with respect to the earth but, in consequence of the rotation of the earth, turns about the vertical through the point of suspension. In the northern hemisphere the plane of swing deviates in the clockwise direction, in the southern hemisphere in the opposite direction. It can be shown [1]) that for an idealized case (very long pendulum, very small amplitude) the time T' in which the plane of swing rotates 360° is given by the formula:

$$T' = \frac{T}{\sin \varphi}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which T is the period of revolution of the earth and φ is the geographic latitude. At the poles $T' = T$, while at the equator $T' = \infty$, i.e. there is no rotation of the plane. In New York ($\varphi = 40°45'$), time $T' = 36$ hours 50 minutes. It further appears from the theory that the angular velocity with which the plane rotates with respect to the meridian is constant.

[1]) See e.g. A. Sommerfeld, Vorlesungen über theoretische Physik, Part I, Dieterich Verl., Wiesbaden 1952.

Fig. 1. The entrance hall in the United Nations building in New York. The Foucault pendulum, a gift from the Netherlands, is mounted above the central column of the staircase.

In practical conditions, the time of rotation is found from the formula:

$$T'' = \frac{T}{\sin\varphi}\left(1 - \frac{3}{8}\frac{a^2}{l^2}\right), \quad . \ . \ . \ . \quad (2)$$

in which a is the amplitude and l the length of the pendulum [2]). In the case of the pendulum in the United Nations building, l is about $17\frac{1}{2}$ m and a about 80 cm. This length gives a period of swing of approximately $8\frac{1}{2}$ sec. The weight of the pendulum bob is about 90 kg.

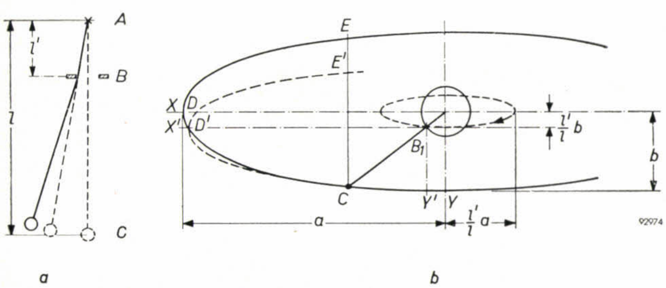
If we try to construct such a pendulum and repeat the celebrated experiment carried out by Foucault in 1851, a number of disturbing effects — due, for example, to imperfections in the rotational symmetry of the wire support — cause the pendulum, after some time, to execute an elliptical or even a circular motion. This had to be avoided with the present pendulum, which was required to swing continuously

[2]) Handbuch der Physik, Part V, p. 339, Springer, Berlin 1927.

without supervision. There were two other technical problems. In the first place there had to be the certainty that the wire would not break after some time of continuous operation, and in the second place a driving system was needed that would be able to provide the pendulum with sufficient energy to compensate for the loss of energy due to air resistance.

A simple method of overcoming the "ellipsing" problem has been described by Charron [3]). At a distance l' below the point of suspension A (see *fig. 2a*) he fixed a ring B having an internal diameter slightly larger than the thickness of the pendulum wire, leaving an annular spacing of d. As soon as the deflection of the bob exceeds the value ld/l', the wire touches the ring and the point of contact then functions as the "point of suspension". The consequence is that the minor axis of an elliptical orbit which may have been forming is rapidly diminished. With reference to fig. 2*b*, this can be roughly explained as follows.

Assuming that the wire, as long as it does not touch the ring, is straight, it will describe at the level of the ring an ellipse geometrically similar to the elliptical orbit of the bob reduced in the ratio l'/l. This is represented in the figure by the small broken ellipse; it is traversed, for example, in the direction of the arrow. The circle represents the limit set by the ring to the deflection of the wire. The moment the wire touches the ring, point B_1 functions as the point of suspension. Assuming that the velocity of the ball in the Y direction is small at that moment, the ball will thereupon proceed to describe the ellipse $CD'E'$, the axes of which lie along the lines B_1X' and B_1Y'. We see, then, that the amplitude



a b

Fig. 2. *a*) Suspension of pendulum according to Charron. The point of suspension is at A. The ring B, whose internal diameter is only slightly larger than the diameter of the wire, prevents the bob C from describing an ellipse.
b) Functioning of Charron's ring. When the pendulum describes an ellipse with semi-axes a and b, the wire touches the ring e.g. at point B_1. If there is sufficient friction between ring and wire, B_1 then functions for a while as the effective point of suspension and the bob describes the ellipse $CD'E'$ instead of the ellipse CDE, which it would describe in the absence of the ring. The amplitude in the Y direction is thus diminished in a half period by the length EE', which is equal to $2bl'/l$.

[3]) F. Charron, Bull. Soc. Astr. Fr. **45**, 457, 1931.

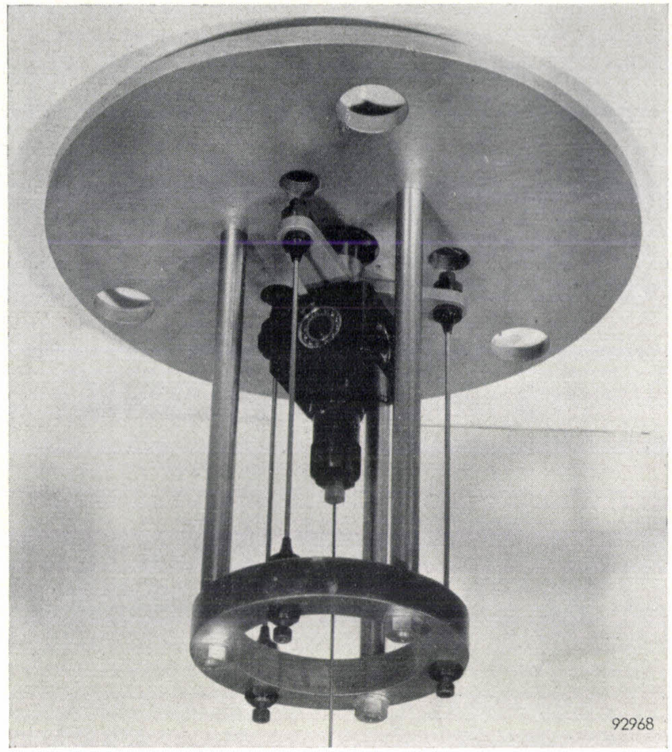


Fig. 3. The actual suspension system.

in the Y direction at the beginning of the following half swing has been reduced by the distance EE', which is approximately equal to $2bl'/l$. The relative reduction for each full swing amounts then to $\Delta b/b = 4l'/l$.

It has been tacitly assumed in the foregoing that while the bob describes the orbit $CD'E'$ the wire stays pressed against the ring at point B_1. As a rule the contact friction will not be sufficient for this to occur. In reality, therefore, $\Delta b/b$ is smaller than calculated here, but this does not alter the fact that the mounting of the ring is a very effective means of combating the elliptical motion.

Charron deduced that the use of this construction would slightly shorten the period of rotation and derived the following expression for the period:

$$T''' = \frac{T}{\sin\varphi}\left(1 - \frac{3}{8}\frac{a^2}{l^2} - \frac{4d}{\pi a}\right). \quad . \quad . \quad . \quad (3)$$

For the pendulum installed in New York a suspension was designed, wich, while equivalent to that of Charron, differs considerably in its construction. *Fig. 3* shows a photograph of the suspension and *fig. 4* a diagram indicating its construction. The mounting plate A, fixed on two beams, carries via three rods B a ring C. This ring supports by means of three flexible rods D a three-armed yoke E fitted with a universal joint F. The wire is suspended from F. By the bending of the flexible rods the yoke can move radially over a distance equal to the radial play s between pin G and the periphery of a hole in the mounting plate. The flexible rods can be moved vertically and their length accurately adjusted; in this way the system as a whole can be given the required lateral stiffness and the pin made to lie exactly in the centre of the hole in the position of rest.

If we compare this arrangement with that of fig. 2*a*, we notice that the universal joint corresponds to the point of the wire which, in the Charron system, is at the height of the ring B. The hole in the mounting plate fulfils here the function of the ring, so that in the new construction the play s is equivalent to the play d in the old. The use of the universal joint avoids the friction and bending which the wire undergoes in Charron's construction at the level of the ring, thus greatly reducing the risk of wear or fatigue failure. The three flexible rods, like Charron's wire length AB, bear the weight G' of the pendulum bob etc. and, via the yoke, exert on the joint a centrally directed force which is proportional to the radial displacement. At a displacement u the above-mentioned point of Charron's wire is subjected to a force of magnitude Gu/l'. By appropriately dimensioning the flexible rods in the new construction, the same magnitude of restoring force can be produced.

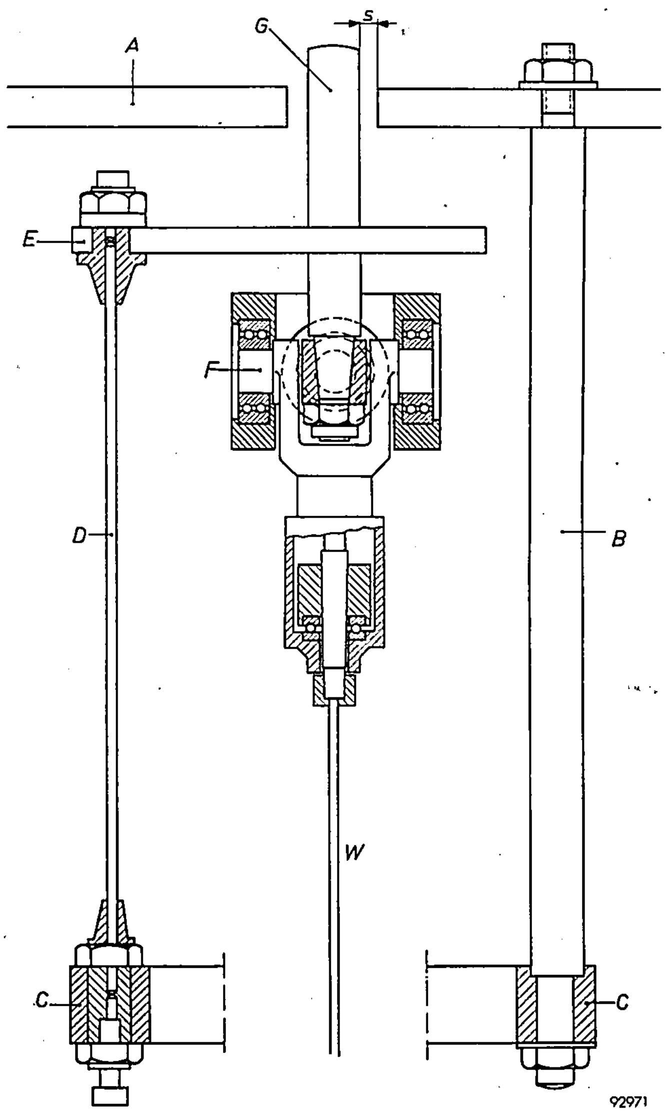


Fig. 4. Schematic diagram of the suspension system of the Foucault pendulum in the United Nations building. Mounting plate A carries three rigid rods B to which a ring C is fixed. Three flexible rods D, clamped on the latter, support a yoke E, which is fitted with a universal joint F. Pin G mounted on E, lies centrally in a hole in the mounting plate, and limits the lateral displacement of the yoke to the small distance s. The suspension wire W is fixed to the joint by clamping in a bifurcated steel cone. The dimensions of the flexible rods are such as to ensure the required lateral stiffness and to ensure that the bending stress in these rods is below their fatigue bending strength.

In connection with the dimensions of the flexible rods it should be borne in mind that the lateral stiffness c is determined not only by the dimensions and the modulus of elasticity but also to a large extent by the compressive force $\frac{1}{3}G'$. The length l' no longer corresponds to any physical quantity such as the length of the flexible rods, but is equal to $G'/3c$.

The lateral stiffness for this case (parallel-constrained ends) is given by the formula

$$c = \frac{P}{l_1}\frac{\frac{1}{2}ql_1}{\tan\frac{1}{2}ql_1 - \frac{1}{2}ql_1}, \quad \ldots\ldots \quad (4)$$

where $q^2 = P/EI$ and $P = \frac{1}{3}G'$. Further, l_1 is the length of the flexible rods, E their modulus of elasticity and I their second moment of area in bending.

The suspension wire is of hard drawn stainless steel with a diameter of 2.5 mm. The wire is clamped at both ends by a truncated cone of hardened steel with internal teeth. The cone has diametric saw cuts at its narrower end and is drawn fast into a ring by the weight of the bob.

In the design of the driving mechanism, which is necessary to keep the pendulum swinging without any reduction in amplitude, account had to be taken of the fact that access to the point of suspension is extremely difficult once the pendulum is mounted. The drive is therefore provided at the bob of the pendulum. The method adopted was to repel the bob by means of eddy currents generated in it by a coil mounted on the pedestal (*fig.* 5), the coil being energized at suitable moments by alternating current. If a copper plate is placed above such a coil at right angles to its axis, voltages will be induced in the plate which are 90° out of phase with the field, and currents which are in their turn almost 90° out of phase with these voltages. The eddy currents are therefore almost 180° out of phase with the current in the coil, thus giving rise to repulsion. If a not too large, horizontal, round plate is placed with its centre-point exactly in the axis of the vertical coil, the resulting force acting on the plate is a vertical one, but if the plate is slightly eccentric to the coil axis, the force will also have a horizontal component which can be used for driving the pendulum. A plate

Fig. 5. The pedestal, designed by architect G. Rietveld, on which the drive coil is mounted. The pedestal bears the inscription:

It is a privilege to live
This day and to-morrow.
Juliana

of this kind is contained in the lower half of the pendulum bob. In order to concentrate the magnetic field on the plate, 18 U-shaped yokes of ferroxcube are fitted around the coil.

This method, then, avoids the use of iron in the ball, and thus prevents the pendulum from being influenced by the magnetic field of the earth or by the steel structure of the building.

Some remarks may be added on the transfer of energy in this driving system. Eddy currents in the copper plate react on the coil with the result that the self-inductance of the latter is smaller than when the plate is absent. When the plate moves away this self-inductance increases and the energy transferred to the plate is approximately equal to $I^2 \Delta L$, where ΔL is the change in the self-inductance and I the current. This does not take into account that I itself changes slightly, but, partly because of the circuit arrangement (see below), this relation holds fairly accurately.

The moment at which the coil is energized is determined by the moment at which the rod in the yoke (*G*, fig. 4) looses contact with the mounting plate [4]). For this purpose, rod and hole were designed to constitute an electrical contact. A delay circuit, which also controls the duration of energization, ensures that the current in the coil is switched on at a specific time after the moment the rod leaves the mounting plate, viz. at approximately the same moment at which the centre of the bob passes the axis of the coil. The anode load of tube *II* in the circuit (see *fig. 6*) includes the D.C. winding of a transductor T [5]). The A.C. winding L_1 of this transductor is connected in series with the drive coil L_2 and a capacitor C''. In addition, another capacitor C' is connected in parallel with L_1. When the D.C. coil of the transductor is not energized, the circuit L_1-C' is in resonance and represents such a high impedance that the current through L_2 is only 85 mA.

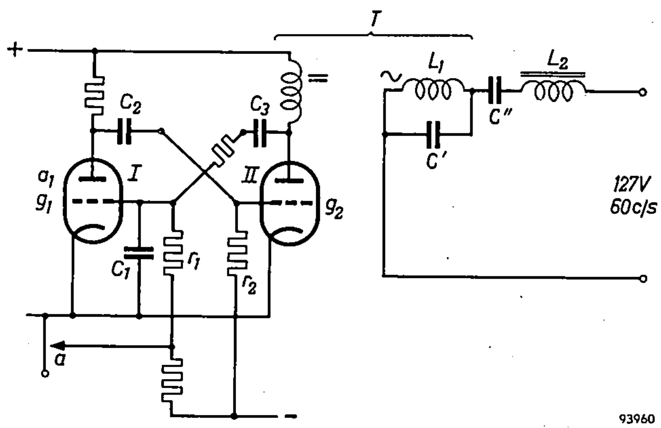


Fig. 6. Control and drive circuit for driving the pendulum. The control circuit (left) causes the required alternating drive current from the mains to be supplied to the drive coil L_2 only after a certain delay after the opening of contact *a*; the circuit sustains the current for a certain fixed time, even if contact *a* is again closed in the meantime. Contact *a* is formed by rod *G* (fig. 4) and the hole in the mounting plate *A*. In the quiescent state, tube *I* is conducting and tube *II* cut off. When *a* opens, the potential of g_1 falls and the potential of a_1 rises: at a certain point the circuit triggers, tube *II* now becoming conducting (via C_2) and tube *I* being cut off. The delay between the opening of *a* and the triggering of the circuit depends on the product C_1r_1. The anode current of tube *II* flows through the D.C. winding of transductor *T* and reduces the value of L_1 such that the drive circuit (right) comes into series resonance. After an interval, which depends on the products C_2r_2 and C_3r_1, the control circuit returns to the quiescent state and L_1 rises again. The value of C' is such that resonance now occurs in the parallel circuit L_1-C'. The latter then represents such a high impedance in the circuit that the current is reduced to an ineffective value.

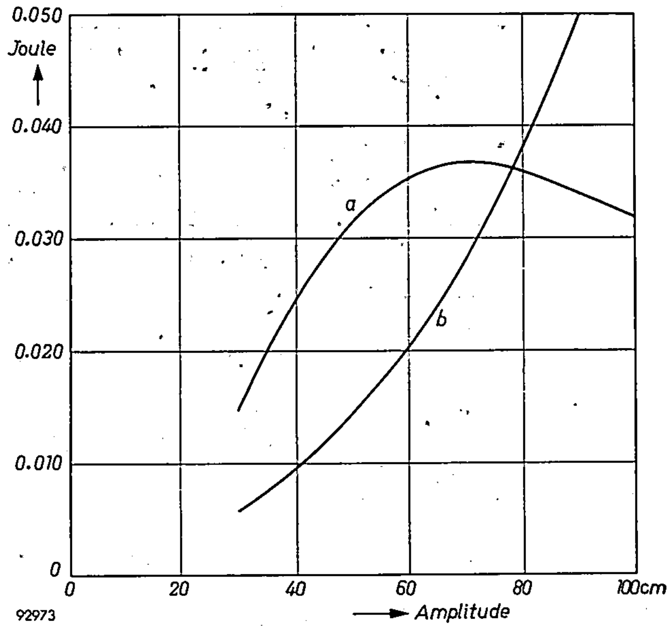


Fig. 7. The energy supplied per period to the pendulum (curve *a*) and that lost by air resistance (curve *b*) as a function of amplitude of swing. At an amplitude of approx. 80 cm the curves intersect. At smaller amplitudes the energy supplied is greater than that lost, i.e. the point of intersection represents a stable state of equilibrium.

When the transductor is energized by the anode current of tube *II*, the self-inductance of L_1 is reduced by about a half. The capacitance of C'' is such that the whole circuit now comes into series resonance, whereupon the current through L_2 rises to 240 mA. The ratio between operating and quiescent currents does not seem particularly large, but it must be remembered that the energy transferred is proportional to the square of the current. In this way about 0.035 joule is supplied to the bob in each period, which is sufficient to provide an amplitude of swing of the required value, viz. approximately 80 cm (see *fig. 7*).

Summary. A description is given of the suspension and driving systems for the Foucault pendulum suspended in the hall of the United Nations building in New York. The pendulum was presented on 7th December 1955 to the Chairman of the General Assembly by the Dutch Foreign Minister H. Luns on behalf of the people of the Netherlands. The main problem of preventing the pendulum bob from describing an ellipse instead of swinging in a flat plane was solved by utilizing the principle described by Charron. The construction is so designed as to reduce to a minimum the risk of the wire breaking owing to wear or fatigue. The drive, which is necessary to compensate for the energy losses, caused mainly by air resistance, is effected by means of a magnetic coil with ferroxcube core placed centrally under the pendulum. This coil is energized by alternating current and induces eddy currents in a copper plate contained inside the lower half of the bob. The energizing current is controlled by an electronic relay, which ensures that the current is switched on some time after the pendulum itself has broken an electrical contact in the suspension system, and keeps it switched on for a certain period. The coil is mounted on a pedestal designed by the architect G. Rietveld.

[4]) R. Stuart Mackay, Amer. J. Phys. **21**, 180, 1953, describes a pendulum driven by eddy currents, the synchronization being derived from the pendulum bob.

[5]) A transductor (saturable reactor) is a choke with an iron core, the self-inductance of which can be varied by changing the magnetization of the iron core with the aid of a D.C. winding.